# Glossary
# UNSW COMP6741

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## Glossary

**acyclic:** A graph is acyclic if it has no cycle as a subgraph.

**bipartite:** A graph $G = (V, E)$ is bipartite if its vertex set can be partitioned into two independent sets. A partition $(A, B)$ of $V$ into independent sets is called a bipartition of $G$. The graph $G$ is then often denoted by $G = (A \uplus B, E)$.

**Boolean formula:** A Boolean formula is constructed from Boolean variables that can take the values true and false (or 1 and 0) by the following operations: conjunction (AND, $\wedge$), disjunction (OR, $\vee$), and negation (NOT, $\neg$).

**clique:** A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a clique in $G$ if $G[S]$ is a complete graph.

**closed neighborhood:** The closed neighborhood of a vertex $v$ in a graph $G$ is $N_G[v] := \{v\} \cup N_G(v)$. The subscript may be omitted if $G$ is clear from the context.

**closed set neighborhood:** The closed neighborhood of a subset of vertices $S \subseteq V$ in a graph $G = (V, E)$ is $N_G[S] := \bigcup_{v \in S} N_G[v]$. The subscript may be omitted if $G$ is clear from the context.

**coloring:** A coloring of a graph $G = (V, E)$ is a function from $V$ to a set of colors (integers) such that every two adjacent vertices in $G$ are mapped to different colors. A $k$-coloring is a coloring using exactly $k$ colors.

**complete:** A graph $G$ is complete if there is an edge between each pair of vertices in $G$. A complete graph on $n$ vertices is denoted by $K_n$.

**Conjunctive Normal Form:** A Boolean formula is in Conjunctive Normal Form if it is a conjunction of clauses, each clause is a disjunctions of literals, and each literal is a Boolean variable or its negation..

**connected:** A graph $G$ is connected if there is a walk between every two vertices of $G$.

**connected component:** Maximal connected subgraph.

**cycle:** 2-regular connected graph. A cycle on $n$ vertices is denoted $C_n$.

**degree:** The degree of a vertex $v$ in a graph $G$ is $d_G(v) := |N_G(v)|$. The subscript may be omitted if $G$ is clear from the context. The degree of a vertex $v$ in a **multigraph:** $G$ is the number of times $v$ appears as an end point of an edge in $E$..

**directed acyclic graph:** A directed acyclic graph (DAG) is a directed graph that contains no directed cycle as a directed subgraph.

**directed cycle:** Orientation of a cycle where each vertex has in-degree 1.

**directed graph:** A directed graph $G$ is an ordered pair $(V, A)$ of a set $V$ of vertices and a set $A$ of arcs, where $A$ is a set of ordered pairs of vertices. Its vertex set is $V(G) = V$ and its arc set is $A(G) = A$.

**directed path:** Orientation of a path where each vertex has in-degree 1, except the start vertex, which has in-degree 0 and out-degree 1.

**disjoint union:** For two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ with $V_1 \cap V_2 = \emptyset$, the disjoint union of $G_1$ and $G_2$, denoted $G_1 \oplus G_2$ is the graph $(V_1 \cup V_2, E_1 \cup E_2)$; in case $V_1 \cap V_2 \neq \emptyset$, vertices need to be renamed before we can take the disjoint union of these graphs.

**distance:** In a graph $G$, the distance between two vertices $u \in V$ and $v \in V$ is the length of the shortest walk minus one between $u$ and $v$, that is the minimum number of edges needed to be traversed to reach $v$ from $u$ and it is denoted by $dist_G(u, v)$.

**dominating set:** A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a dominating set of $G$ if $N_G[S] = V$.

**feedback vertex set:** A feedback vertex set of a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that $G - S$ is acyclic.

**forest:** Acyclic graph.

**graph:** A (simple, undirected) graph $G$ is an ordered pair $(V, E)$ of a set $V$ of vertices and a set $E$ of edges, where $E$ is a set of unordered pairs of distinct vertices. Its vertex set is $V(G) = V$ and its edge set is $E(G) = E$.

**in-degree:** The in-degree of a vertex $v$ in a directed graph $D$ is $d_D^-(v) := |\{uv \in A\}|$. The subscript may be omitted if $D$ is clear from the context.

**independent set:** A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is an independent set of $G$ if $G[S]$ has no edges.

**induced subgraph:** For a graph $G = (V, E)$ and a vertex set $S \subseteq V$, the subgraph of $G$ induced on $S$ is the graph $G[S] := (S, \{uv \in E : u, v \in S\})$.

**maximal (set):** For a set $\mathcal{S}$ of subsets of a ground set $U$, a set $X \in \mathcal{S}$ is maximal if there exists no set $Y \in \mathcal{S}$ with $X \subsetneq Y$.

**maximum (set):** For a set $\mathcal{S}$ of subsets of a ground set $U$, a set $X \in \mathcal{S}$ is maximum if there exists no set $Y \in \mathcal{S}$ with $|Y| > |X|$.

**maximum degree:** The maximum degree of a graph $G = (V, E)$ is $\Delta(G) := \max_{v \in V} d_G(v)$.

**minimum degree:** The minimum degree of a graph $G = (V, E)$ is $\delta(G) := \min_{v \in V} d_G(v)$.

**multigraph:** A multigraph $G$ is an ordered pair $(V, E)$ of a set $V$ of vertices and a *multi*set $E$ of edges, where $E$ is a multiset of unordered pairs of vertices. Its vertex set is $V(G) = V$ and its edge set is $E(G) = E$.

**open neighborhood:** The (open) neighborhood of a vertex $v$ in a graph $G = (V, E)$ is $N_G(v) := \{u \in V : uv \in E\}$. The subscript may be omitted if $G$ is clear from the context.

**open set neighborhood:** The (open) neighborhood of a subset of vertices $S \subseteq V$ in a graph $G = (V, E)$ is $N_G(S) := N_G[S] \setminus S$. The subscript may be omitted if $G$ is clear from the context.

**order of growth:** Let $g : \mathbb{R}_+ \to \mathbb{R}_+$ be a function. The set $O(g(n))$ contains every function $f$ such that there exist $c, n_0 \geq 0$ such that $f(n) \leq c \cdot g(n)$ for every $n \geq n_0$. The set $o(g(n))$ contains every function $f$ such that for every $\epsilon > 0$ there exists a $n_0 \geq 0$ such that $f(n) \leq \epsilon \cdot g(n)$ for every $n \geq n_0$. For the set $\Omega(g(n))$, we have that $f(n) \in \Omega(g(n))$ iff $g(n) \in O(f(n))$. For the set $\omega(g(n))$, we have that $f(n) \in \omega(g(n))$ iff $g(n) \in o(f(n))$. For the set $\Theta(g(n))$, we have that $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.

**orientation:** An orientation of a graph $G$ is a directed graph $D$ that has exactly one arc for each edge of $G$ with the same endpoints.

**out-degree:** The out-degree of a vertex $v$ in a directed graph $D$ is $d_D^+(v) := |\{vu \in A\}|$. The subscript may be omitted if $D$ is clear from the context.

**path:** Tree with maximum degree at most 2. A path on $n$ vertices is denoted $P_n$.

**regular:** A graph is $d$-regular if each of its vertices has degree $d$. A graph is regular if it is $d$-regular for some $d$.

**subgraph:** A graph $H$ is a subgraph of a graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

**tree:** Acyclic, connected graph.

**vertex cover:** A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a vertex cover of $G$ if each edge of $G$ is incident to at least one vertex of $S$.

**vertex removal:** For a graph $G = (V, E)$ and a vertex set $S \subseteq V$, the graph obtained by removing $S$ from $G$ is $G - S := G[V \setminus S]$. If $S = \{u\}$, we may write $G - u$ instead of $G - \{u\}$.

**walk:** Sequence of vertices in a graph, with each vertex being adjacent to the vertices immediately preceding and succeeding it in the sequence.

## Problem Definitions

$k$-COLORING

Given a graph $G$, determine if there is a coloring of $G$ with at most $k$ colors.

$k$-SAT

Given a Boolean formula in Conjunctive Normal Form where each clause has at most $k$ literals, determine if there is an assignment of its variables such that the formula evaluates to `true`.

DOMINATING SET

Given a graph $G$ and an integer $k$, determine whether $G$ has a dominating set of size $k$.

FEEDBACK VERTEX SET

Given a (multi)graph $G$ and an integer $k$, determine whether $G$ has a feedback vertex set of size at most $k$.

INDEPENDENT SET

Given a graph $G$ and an integer $k$, determine whether $G$ has an independent set of size $k$.

MAXIMUM INDEPENDENT SET

Given a graph $G$, find an independent set of $G$ of maximum cardinality.

Minimum Vertex Cover

Given a graph $G$, find a vertex cover of $G$ of minimum cardinality.

Sat

Given a Boolean formula, determine if there is an assignment of its variables such that the formula evaluates to true.

Traveling Salesman Problem

Given a set $\{1, \ldots, n\}$ of $n$ cities, the distance $d(i, j)$ between every two cities $i$ and $j$, and an integer $k$, determine whether there is a tour with total distance at most $k$. A *tour* is a permutation of the cities starting and ending in city 1.

Vertex Cover

Given a graph $G$ and an integer $k$, determine whether $G$ has a vertex cover of size $k$.